TECHNICAL APPENDIX

A. Non-stochastic steady state for the model *without* credit market imperfections

$$P = \frac{\upsilon}{\upsilon - 1} MC$$

$$W^A = W^B = W = (1-\alpha)\left(\frac{\alpha}{P}\right)^{\frac{\alpha}{1-\alpha}}$$

$$L \equiv L^A + L^B = \sqrt{\frac{\upsilon - 1}{\left(\upsilon + \frac{\alpha}{1-\alpha}\right)\gamma(1-\gamma)\kappa}} \qquad L^A = \gamma L \qquad L^B = (1-\gamma)L$$

$$K \equiv K^A + K^B = L\left(\frac{\alpha}{P}\right)^{\frac{1}{1-\alpha}} \qquad K^A = \gamma K \qquad K^B = (1-\gamma)K$$

$$Y \equiv Y^A + Y^B = L\left(\frac{\alpha}{P}\right)^{\frac{\alpha}{1-\alpha}} \qquad Y^A = \gamma Y \qquad Y^B = (1-\gamma)Y$$

$$C = \frac{WL}{P}\left(\frac{\upsilon + \frac{\alpha}{1-\alpha}}{\upsilon - 1}\right)$$

$$\Pi = \frac{PL}{\upsilon}\left[\left(\frac{\alpha}{P}\right)^{\frac{1}{1-\alpha}} + \left(\frac{W}{P}\right)\left(\frac{\upsilon + \frac{\alpha}{1-\alpha}}{\upsilon - 1}\right)\right]$$

B. Solution for the log-linear model *without* credit market imperfections

$$\hat{W}_t^i = \frac{\hat{S}_t^i + \hat{X}_t^i - \alpha \hat{P}_t}{1-\alpha}, \qquad i = A, B$$

$$\hat{C}_t = \frac{\widehat{EPI}_t - \hat{P}_t}{1-\alpha} - \left(\frac{1}{2} + \frac{g\upsilon}{2(\upsilon-1)}\right)\left(\widehat{MC}_t - \hat{P}_t\right) = \left(\frac{1+\alpha}{2(1-\alpha)} - \frac{g\upsilon}{2(\upsilon-1)}\right)\left(\widehat{NEER}_t - \hat{P}_t\right)$$

$$\hat{\Pi}_t = \hat{P}_t + \frac{1}{f}\left[\frac{f}{1-\alpha}\left(\widehat{EPI}_t - \hat{P}_t\right) + (f-2)\left(\frac{1}{2} + \frac{g\upsilon}{2(\upsilon-1)}\right)\left(\widehat{MC}_t - \hat{P}_t\right)\right] =$$

$$= \hat{P}_t + \frac{1}{f}\left[\frac{f}{1-\alpha} + (f-2)\left(\frac{1}{2} + \frac{g\upsilon}{2(\upsilon-1)}\right)\right]\left(\widehat{NEER}_t - \hat{P}_t\right)$$

$$\hat{L}_t = \gamma \hat{L}_t^A + (1-\gamma)\hat{L}_t^B = \left(\frac{1}{2} + \frac{g\upsilon}{2(\upsilon-1)}\right)\left(\widehat{MC}_t - \hat{P}_t\right) = \left(\frac{1}{2} + \frac{g\upsilon}{2(\upsilon-1)}\right)\left(\widehat{NEER}_t - \hat{P}_t\right)$$

$$\hat{K}_t = \gamma \hat{K}_t^A + (1-\gamma)\hat{K}_t^B = \frac{\widehat{EPI}_t - \hat{P}_t}{1-\alpha} - \left[\frac{1}{2} + \frac{g\upsilon}{2(\upsilon-1)}\right]\left(\widehat{MC}_t - \hat{P}_t\right) = \left(\frac{3-\alpha}{2(1-\alpha)} + \frac{g\upsilon}{2(\upsilon-1)}\right)\left(\widehat{NEER}_t - \hat{P}_t\right)$$

$$\hat{Y}_t = \gamma\hat{Y}_t^A + (1-\gamma)\hat{Y}_t^B = \frac{\alpha\left(\widehat{EPI}_t - \hat{P}_t\right)}{1-\alpha} + \left(\frac{1}{2} + \frac{g\upsilon}{2(\upsilon-1)}\right)\left(\widehat{MC}_t - \hat{P}_t\right) = \left(\frac{1+\alpha}{2(1-\alpha)} + \frac{g\upsilon}{2(\upsilon-1)}\right)\left(\widehat{NEER}_t - \hat{P}_t\right) \quad \textbf{(19)}$$

Above, $\widehat{EPI}_t \equiv \gamma\left(\hat{S}_t^A + \hat{X}_t^A\right) + (1-\gamma)\left(\hat{S}_t^B + \hat{X}_t^B\right)$ is the export price index. I also repeatedly substituted for $\widehat{MC}_t$ using equation **(6)**.

C. Non-stochastic steady state for the model *with* credit market imperfections

The solution for the non-stochastic steady state of the model of Section 5.1 assumes the normalization $S^i = M^i = X^i = 1$, for $i = A, B$, and is as follows:

$$P = \frac{\upsilon}{\upsilon-1}MC$$

$$W^A = W^B = (1-\alpha)\left(\frac{\alpha\delta}{P}\right)^{\frac{\alpha}{1-\alpha}}$$

$$L \equiv L^A + L^B = \sqrt{\frac{\upsilon-1}{\left(\upsilon + \frac{\alpha\delta}{1-\alpha}\right)\gamma(1-\gamma)\kappa}} \qquad L^A = \gamma L \qquad L^B = (1-\gamma)L$$

$$K \equiv K^A + K^B = L\left(\frac{\alpha\delta}{P}\right)^{\frac{1}{1-\alpha}} \qquad K^A = \gamma K \qquad K^B = (1-\gamma)K$$

$$Y \equiv Y^A + Y^B = L\left(\frac{\alpha\delta}{P}\right)^{\frac{\alpha}{1-\alpha}} \qquad Y^A = \gamma Y \qquad Y^B = (1-\gamma)Y$$

$$C = \frac{WL}{P}\left(\frac{\upsilon + \frac{\alpha\delta}{1-\alpha}}{\upsilon-1}\right)$$

$$\Pi = \frac{PL}{\upsilon}\left[\left(\frac{\alpha\delta}{P}\right)^{\frac{1}{1-\alpha}} + \left(\frac{1-\alpha}{P}\right)\left(\frac{\alpha\delta}{P}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{\upsilon + \frac{\alpha\delta}{1-\alpha}}{\upsilon-1}\right)\right]$$

$$\frac{R}{P} = \frac{1}{\delta}$$

$$1+\rho = \frac{1}{\delta(1+i^*)}$$

$$B = PK\left[1 - \left[\delta(1+i^*)\right]^{\frac{1}{\mu}}\right]$$

$$N = PK\left[\delta(1+i^*)\right]^{\frac{1}{\mu}}$$

Note that we need $1/\delta > 1+i^*$ in order to have $\rho > 0$ in steady state. Note also that the steady-state gross markup R/P charged by entrepreneurs to firms equals the inverse of entrepreneurs' saving rate.

D. The log-linear model with credit market imperfections and its solution

I log-linearize equations **(2)-(3)**, **(11-A), (14), (17)-(18)**, **(20)-(23)** around the non-stochastic steady state:

$$\hat{P}_t + \hat{C}_t = \gamma(1-f)\left(\hat{W}_t^A + \hat{L}_t^A\right) + (1-\gamma)(1-f)\left(\hat{W}_t^B + \hat{L}_t^B\right) + f\hat{\Pi}_t, \qquad \text{where } f = \frac{1-\alpha+\alpha\delta}{\upsilon(1-\alpha)+\alpha\delta}$$

$$-\hat{C}_t - \hat{P}_t = \hat{L}_t^i - \hat{W}_t^i, \qquad i = A,B$$

$$f\hat{\Pi}_t = \hat{P}_t + \hat{C}_t - \frac{\upsilon-1}{\upsilon}\left(\widehat{MC}_t + \hat{C}_t\right) + \left(f + g - \frac{1}{\upsilon}\right)\left(\hat{P}_t + \hat{K}_{t+1}\right) - g\left(\widehat{MC}_t + \hat{K}_{t+1}\right), \qquad \text{where } g = \frac{\alpha\delta(\upsilon-1)^2}{\upsilon\left[\upsilon(1-\alpha)+\alpha\delta\right]}$$

$$\hat{Y}_t^i = (1-\alpha)\hat{L}_t^i + \alpha\hat{K}_t^i, \qquad i = A,B$$

$$\hat{S}_t^i + \hat{X}_t^i + \hat{Y}_t^i = \hat{W}_t^i + \hat{L}_t^i, \qquad i = A,B$$

$$\hat{S}_t^i + \hat{X}_t^i + \hat{Y}_t^i = \hat{R}_t + \hat{K}_t^i, \qquad i = A,B$$

$$\hat{N}_t + (a_1 - 1)\left(\hat{S}_t^A + \hat{B}_{t+1}\right) = a_1\left(\hat{P}_t + \hat{K}_{t+1}\right), \qquad \text{where } a_1 = \left[\delta(1+i^*)\right]^{-\frac{1}{\mu}}$$

$$(1+\hat{\rho}_{t+1}) = \mu\left(\hat{P}_t + \hat{K}_{t+1} - \hat{N}_t\right)$$

$$\hat{N}_t = a_1\left(\hat{R}_t + \hat{K}_t\right) - (a_1 - 1)\left[(1+\hat{\rho}_t) + \hat{S}_t^A + \hat{B}_t\right]$$

$$E_t\left(\hat{R}_{t+1}\right) = \hat{P}_t + E_t\left(\hat{S}_{t+1}^A\right) - \hat{S}_t^A + \left(1+\hat{\rho}_{t+1}\right)$$

In general, $\hat{Z}_t \equiv dZ_t/Z$, with the exception of $(1+\hat{\rho}_t) \equiv \log(1+\rho_t) - \log(1+\rho)$. MC and P are set according to equations **(6)** and **(10)**. The dynamics of $\hat{M}_t^i$ and $\hat{X}_t^i$ are given by equations **(12)-(13)** and **(15)-(16)**. The government sets the exchange rates S^i. In the above system, $\hat{B}_t$, $\hat{K}_t$, and $(1+\hat{\rho}_t)$ are predetermined variables.

Most of the variables of the system can be easily substituted out:

$$\hat{W}_t^i = \frac{\hat{S}_t^i + \hat{X}_t^i - \alpha\hat{R}_t}{1-\alpha}, \qquad i = A,B$$

$$\hat{L}_t = \gamma\hat{L}_t^A + (1-\gamma)\hat{L}_t^B = \frac{\widehat{EPI}_t - \alpha\hat{R}_t}{1-\alpha} - \hat{P}_t - \hat{C}_t,$$

where the export price index (EPI) is defined as: $\widehat{EPI}_t \equiv \gamma\left(\hat{S}_t^A + \hat{X}_t^A\right) + (1-\gamma)\left(\hat{S}_t^B + \hat{X}_t^B\right)$

$$\hat{Y}_t = \gamma\hat{Y}_t^A + (1-\gamma)\hat{Y}_t^B = \left(\frac{1+\alpha}{1-\alpha}\right)\widehat{EPI}_t - \frac{2\alpha}{1-\alpha}\hat{R}_t - \hat{P}_t - \hat{C}_t$$

$$\hat{C}_t = \frac{\frac{2(1-f)}{1-\alpha}\left(\hat{EPI}_t - \alpha\hat{R}_t\right) - \left(1-2f-g+\frac{1}{\upsilon}\right)\hat{P}_t - \left(\frac{\upsilon-1}{\upsilon}+g\right)\hat{MC}_t - \left(\frac{1}{\upsilon}-f\right)\hat{K}_{t+1}}{\frac{\nu-1}{\nu}+(1-f)}$$

$$(1+\hat{\rho}_t) = \mu\left(\hat{P}_{t-1}+\hat{K}_t-\hat{N}_{t-1}\right)$$

$$\hat{B}_{t+1} = \frac{a_1\left(\hat{P}_t+\hat{K}_{t+1}\right)-\hat{N}_t}{a_1-1} - \hat{S}_t^A$$

For the remaining 3 variables – the rental price of capital *R*, net worth *N*, and the capital stock *K* – one can write the following system of 3 first-order difference equations:

$$\begin{pmatrix} E_t(\hat{R}_{t+1}) \\ \hat{N}_t \\ \hat{K}_{t+1} \end{pmatrix} = A\begin{pmatrix} \hat{R}_t \\ \hat{N}_{t-1} \\ \hat{K}_t \end{pmatrix} + B,$$ where *A* is a (3x3) matrix and *B* is a (3x1) matrix, such that:

$$A(1,1) = \mu[A(3,1)-A(2,1)] \qquad A(1,2) = \mu[A(3,2)-A(2,2)] \qquad A(1,3) = \mu[A(3,3)-A(2,3)]$$

$$A(2,1) = a_1 \qquad A(2,2) = \mu(a_1-1)+1 \qquad A(2,3) = -\mu(a_1-1)$$

$$A(3,1) = \frac{\frac{\upsilon-1}{\upsilon}(1+\alpha)+(1-\alpha)(1-f)}{\left(\frac{1}{\upsilon}-f\right)(1-\alpha)} \qquad A(3,2) = 0 \qquad A(3,3) = \frac{2-\frac{1}{\upsilon}-f}{\frac{1}{\upsilon}-f}$$

$$B(1,1) = E_t\left(\hat{S}_{t+1}^A\right) - \hat{S}_t^A + (1+\mu)\hat{P}_t + \mu[B(3,1)-B(2,1)]$$

$$B(2,1) = -(a_1-1)\left[\left(\mu+\frac{a_1}{a_1-1}\right)\hat{P}_{t-1}+\hat{S}_t^A-\hat{S}_{t-1}^A\right]$$

$$B(3,1) = \frac{-\left(\frac{\upsilon-1}{\upsilon}+g\right)(1-\alpha)\hat{MC}_t - 2\frac{\upsilon-1}{\upsilon}\hat{EPI}_t + (1-\alpha)\left(1+f+g-\frac{2}{\upsilon}\right)\hat{P}_t}{\left(\frac{1}{\upsilon}-f\right)(1-\alpha)}$$

To compute the rational expectations equilibrium for this system, I followed the method outlined in Monacelli and Natalucci (2002), which is a simplified version of Blanchard and Kahn (1980). Of the 3 eigen values I computed for matrix *A*, using the parameter values of Section 5.2, one is outside and two are inside the unit circle. Given that I have one unstable (*R*) and two stable (*N* and *K*) variables in the system, the solution is unique.

E. Log-linearizing equation (23)

Equation **(23)** is the hardest one to log-linearize because it contains on its left-hand side the expectation of a non-linear function of two random variables. Because of Jensen's inequality, a first-order Taylor approximation is inappropriate. I use a trick from Obstfeld and Rogoff (1996, p. 504), where they show how to linearize a stochastic Euler equation.

The right-hand side is easy – just take logs to get:

$$\log P_t - \log S_t^A + \log\left(1+i^*\right) + \log\left(1+\rho_{t+1}\right)$$

For the left-hand side, assume that the random variable R_{t+1}/S_{t+1}^A is lognormally distributed with a conditional variance which is constant over time. This is consistent with the assumption (adopted in Section 5.2) of Normal shocks to $\hat{S}_t^{BA}$. Then:

$$E_t\left(\frac{R_{t+1}}{S_{t+1}^A}\right) = E_t\left[\exp\left(\log\frac{R_{t+1}}{S_{t+1}^A}\right)\right] = \exp\left[E_t\left(\log R_{t+1}\right) - E_t\left(\log S_{t+1}^A\right) + \frac{1}{2}Var\left(\log\frac{R_{t+1}}{S_{t+1}^A}\right)\right]$$

Take logs to get:

$$E_t\left(\log R_{t+1}\right) - E_t\left(\log S_{t+1}^A\right) + \frac{1}{2}Var\left(\log\frac{R_{t+1}}{S_{t+1}^A}\right)$$

Since I am interested in the system's dynamic response to shocks, and not in trend movements, I omit (as do Obstfeld and Rogoff) the constant variance term. In logs, equation **(23)** is approximated by:

$$E_t\left(\log R_{t+1}\right) - E_t\left(\log S_{t+1}^A\right) = \log P_t - \log S_t^A + \log\left(1+i^*\right) + \log\left(1+\rho_{t+1}\right)$$

Re-arrange to get:

$$E_t\left(\log R_{t+1}\right) = E_t\left(\log S_{t+1}^A\right) - \log S_t^A + \log P_t + \log\left(1+i^*\right) + \log\left(1+\rho_{t+1}\right)$$

Use the steady-state relationship:

$$\frac{R}{P} = \frac{1}{\delta} = (1+i^*)(1+\rho) \qquad \Leftrightarrow \qquad \log R = \log P + \log(1+i^*) + \log(1+\rho)$$

Subtract the log equation above from the preceding one to finally arrive at:

$$E_t\left(\hat{R}_{t+1}\right) = E_t\left(\hat{S}_{t+1}^A\right) - \hat{S}_t^A + \hat{P}_t + \widehat{\left(1+\rho_{t+1}\right)}$$

Note that ρ_{t+1} is pre-determined at time t. Finally, this equation can be re-arranged as follows:

$$E_t\left(\hat{R}_{t+1}\right) - \hat{P}_t = E_t\left(\hat{S}_{t+1}^A\right) - \hat{S}_t^A + \widehat{\left(1+\rho_{t+1}\right)}$$

One can think of the ratio R_t/P_t as the domestic-currency, risk-adjusted return on investment. It is the equivalent to the domestic nominal interest rate in the model. The dynamics of R_t/P_t are traced out in Figures 3-5.